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A Comparison of Design Variables for Control Theory Based Airfoil Optimization

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Introduction

This paper describes the implementation of optimization techniques based on control theory for airfoil design. In our previous work in the area [6, 7, 19, 11] it was shown that control theory could be employed to devise effective optimization procedures for two-dimensional profiles by using either the potential flow or the Euler equations with either a conformal mapping or a general coordinate system. We have also explored three-dimensional extensions of these formulations recently [10, 9, 20]. The goal of our present work is to demonstrate the versatility of the control theory approach by designing airfoils using both Hicks-Henne functions and B-spline control points as design variables. The research also demonstrates that the parameterization of the design space is an open question in aerodynamic design.

Formulation of the design problem as a control problem

In this research control theory serves to provide computationally inexpensive gradient information to a standard numerical optimization method. For flow about an airfoil or wing, the aerodynamic properties which define the cost function are functions of the flow-field variables (w) and the physical location of the boundary, which may be represented by the function $\mathcal{F}$, say. Then

$$I = I(w, \mathcal{F})$$

and a change in $\mathcal{F}$ results in a change

$$\delta I = \frac{\partial I^T}{\partial w}\delta w + \frac{\partial I^T}{\partial \mathcal{F}}\delta \mathcal{F} \tag{1}$$

in the cost function. Each term in (1), except for δw, can be easily obtained. Finite difference methods evaluate the gradient by making a small change in each design variable separately, and then recalculate both the grid and flow-field variables. This requires a number of additional flow calculations equal to the number of design variables. Using control theory, the governing equations of the flowfield are introduced as a constraint in such a way that the final evaluation of the gradient does not require multiple flow solutions. In order to achieve this, δw must be eliminated from (1). The governing equation R and its first variation express the

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dependence of w and $\mathcal{F}$ within the flowfield domain D:

$$R(w, \mathcal{F}) = 0, \quad \delta R = \left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} = 0. \tag{2}$$

Next, introducing a Lagrange multiplier ψ, we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w} \delta w + \frac{\partial I^T}{\partial \mathcal{F}} \delta \mathcal{F} - \psi^T \left(\left[\frac{\partial R}{\partial w}\right] \delta w + \left[\frac{\partial R}{\partial \mathcal{F}}\right] \delta \mathcal{F} \right) \\ &= \left\{ \frac{\partial I^T}{\partial w} - \psi^T \left[\frac{\partial R}{\partial w}\right] \right\} \delta w + \left\{ \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right] \right\} \delta \mathcal{F} \end{aligned}$$

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w} \tag{3}$$

the first term is eliminated, and we find that the desired gradient is given by

$$\mathcal{G}^T = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right]. \tag{4}$$

The advantage is that (4) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations. The main cost is in solving the adjoint equation (3). In general, the adjoint problem is about as complex as a flow solution. If the number of design variables is large, the cost differential between one adjoint solution and the large number of flowfield evaluations required to determine the gradient by finite differencing becomes compelling. Once equation (4) is obtained, $\mathcal{G}$ can be fed into any numerical optimization algorithm to obtain an improved design.

Issues of importance for design problems

The development of aerodynamic design proceedures that employ an adjoint equation formulation is currently being investigated by many researchers. These methods promise to allow computational fluid dynamics methods to become true aerodynamic design methods. References [1, 2, 12, 13, 16, 15, 14, 21, 17, 22] represent a partial list of recent works that reflect this developing field. However, as is the case in any new research field, many questions remain. Probably the most salient issues of concern are the following:

1. Discrete vs. continuous sensitivities
2. Choice of optimization procedures
3. Treatment of geometric and aerodynamic constraints
4. The level of coupling between design and analysis
5. The parameterization of the design space

These topics, which were outlined in a proposal to the U.S. Airforce in 1991 [8], still require intensive investigation in the up coming years. With regards to the first item, it is historically interesting that Jameson in 1988 [6] first developed the equations for a continuous approach to transonic airfoil and wing design, whereby the steps represented by equations (1-4) were applied to the governing differential equations. The resulting adjoint differential equations with the appropriate boundary conditions may then be discretized and solved in a manner similar to that used for the flow solver. One may alternatively derive a set of discrete adjoint equations directly from the discrete approximation to the flow equations by following the procedure outlined in equations (1-4). The resulting discrete adjoint equations are one of the possible discretizations of the continuous adjoint equations. This alternative is mentioned in [7], but was not adopted in that work because of the complexity of the resulting discrete adjoint system. It has the advantage that the resulting discrete gradients are the true gradients of the discrete model of the flow equations, and it has found favor in the work of Taylor et al. [16, 13, 12] and others [1, 2]. It seems that both alternatives have some advantages. The continuous approach gives the researcher some hope for an intuitive understanding of the adjoint system and

its related boundary conditions. It also permits an easy recycling of the solution methodology used for the flow solver. The discrete approach, in theory, maintains perfect algebraic consistency at the discrete level. If properly implemented it will give gradients which closely match those obtained through finite differences. The continuous formulation produces slightly inaccurate gradients due to differences in the discretization. However, these inaccuracies must vanish as the mesh width is reduced. A drawback of the discrete approach is that once the large matrix problem of equation (3) is defined it becomes more difficult to recycle the flow solution algorithm on the resulting linear algebra problem. It is subject, moreover, to the difficulty that the discrete flow equations often contain nonlinear flux limiting functions which are not differentiable. It also limits the flexibility to use adaptive discretization techniques with order and mesh refinement, such as the h-p method, because the adjoint discretization is fixed by the flow discretization

Design variables

It turns out that the determination of items (2-4) in the above list strongly hinge on the choice for (5). In Jameson's first works in the area [6, 7, 10], every surface mesh point was used as a design variable. In three-dimensional wing design cases this led to as many as 4224 design variables [10] The use of the adjoint method eliminated the unacceptable costs that such a large number of design variables would incur for traditional finite difference methods. If the approach were extended to treat complete aircraft configurations, at least tens of thousands of design variables would be necessary. Such large numbers of design variables preclude the use of descent algorithms such as Newton or quasi-Newton approaches simply because of the high cost of matrix operations for such methods. The limitation of using a simple descent procedure such as steepest descent has the consequence that significant errors can be tolerated initially in the gradient evaluation. Therefore such methods favor tighter coupling of the flow solver, the adjoint solver, and the overall design problem to accelerate convergence. Ta'asan et al [21, 14] took advantage of this by formulating the design problem as a one shot procedure where all three systems are advanced simultaneously. Choosing such a design space suffers from admitting poorly conditioned design problems. This is best exemplified by the case where only one point on the surface of an airfoil is moved, resulting in a highly nonlinear design response. In his original work Jameson [7, 9] treated this poor conditioning of the design problem by smoothing the control (surface shape) and thus removing the problematic high frequency content from the advancing solution shape.

Further, arbitrarily freeing all surface points as part of the design makes it difficult to enforce geometric constraints, such as maintaining fuel volume, without resorting to constrained optimization algorithms. Nevertheless, this choice for variables remains attractive since it admits the greatest possible design space.

Ray Hicks and others [5, 4, 18] have in the past parameterized the design space using sets of smooth functions that either generate or perturb the initial geometry. By using such a parameterization it is possible to work with considerably fewer design variables than the choice of every mesh point. Hicks initially adopted the approach because his work in the past exclusively used finite difference based gradient design methods that are inherently intolerant of more than a few dozen design variables for large problems. However, since these early methods had to content themselves with at most a hundred design variables, great freedom in the choice of the design algorithm was possible. One simple choice of design variables for airfoils suggested by Hicks and Henne [4], have the following "sine bump" form:

$$b(x) = \left[\sin\left(\pi x^{\frac{\log(.5)}{\log(t_1)}}\right)\right]^{t_2}, \quad 0 \le x \le 1$$

Here t_1 locates the maximum of the bump in the range $0 \le x \le 1$ at $x = t_1$, since the maximum occurs when $x^{\alpha} = \frac{1}{2}$, where $\alpha = \log\frac{1}{2}/\log t_1$, or $\alpha \log t_1 = \log\frac{1}{2}$. Parameter t_2 controls the width of the bump.

When distributed over the entire chord on both upper and lower surfaces, these analytic perturbation functions admit a large possible design space. They can be chosen such that symmetry, thickness, or volume can be explicitly constrained, thus avoiding the use of expensive constrained optimization algorithms to address geometric constraints. Further, particular choices of these variables will concentrate the design effort in regions where refinement is needed, while leaving the rest of the airfoil section virtually undisturbed. The disadvantage of these functions is that they are not orthogonal, and there is no simple way to form a basis from these functions which is complete for the space of continuous functions which vanish at $x = 0$ and $x = 1$. Thus, they do not guarantee that a solution, for example, of the inverse problem for a realizable target pressure distribution will necessarily be attained. Nevertheless, they have proved to be quite effective in realizing design improvements with a limited number of design variables. The design process that uses these basis spaces can be accelerated toward convergence by either tighter coupling of the individual design elements, as was the case when using the mesh points themselves, or through the use of higher order optimization algorithms. Finally they have

one last advantage over using the mesh points; there is no need to smooth the resulting solutions as the design proceeds when using Hicks-Henne functions, since by construction higher frequecies are not admitted and thus the design spaces are naturally well posed

Another set of design variables that has recently achieved favor is the use of B-spline control points. In this approach the geometry is defined by B-spline curves or surfaces while their control points become the design variables. Like the Hicks-Henne functions, B-splines allow for a greatly reduced number of design variables, and thus permit the use of, say, a quasi-Newton design procedure. If the upper and lower surfaces of an airfoil are separated, the method easily admits camber or thickness constraints explicitly within the design space. Further, local control is also possible by choosing only a limited number of control points as active design variables. This method in practice seems to have an advantage over the Hicks-Henne functions in that a more complete basis space of admitted airfoils is permitted with the same number of design variables. Finally since these curves and surfaces are the natural entities used in CAD enviroments, they provide a straightforward way of integrating CAD and aerodynamic design.

Numerical Experiments

In this short paper, a study of both Hicks-Henne and B-spline control points is explored to begin to address the open issue presented in item (5) above. For the details of the development and implementation of the full design procedure, the reader is referred to our previous works listed earlier. Here the two-dimensional airfoil design problem is accomplished by combining an Euler flow solver, a corresponding adjoint solver, and an unconstrained quasi-Newton optimization algorithm that uses BFGS updates [3, 11]. The method has been demonstrated to be successful in the design of various airfoils at transonic conditions and with various objective functions.

Figure (1) shows a design case where 25 Hicks-Henne bumps are used as design variables for both the upper and lower surfaces. The bumps are arranged with high clustering toward the leading and trailing edges to give adequate design fidelity in these sensitive areas. Figure (1a) shows the initial conditions for an inverse design problem where a target pressure distribution for an RAE 2822 airfoil at $\alpha = 1°$, $M = .75$, and $C_l = .6030$ is specified and a NACA 0012 airfoil is prescribed as a starting airfoil. Both the target and the initial airfoil show strong shocks in their respective solutions. After 5 design iterations significant progress is seen in matching the target pressures. Moreover, the cost function, which is measured as an integral of the square of the differences between the two pressure distributions, has dropped from 68.556 to 4.247. The proper location for the shock has already been achieved. By 10 iterations the cost function has been further reduced to 0.702, and after 30 iterations no differences are visible in the solution where the cost function is only .010.

Figure (2) shows the same design problem treated with two B-spline curves for the upper and lower surfaces. Each curve was defined by 18 control points with a degree of 4 and was chosen to represent the airfoil shape directly. The y ordinate of each control point was used as a design variable. A close approximation to a NACA 0012 airfoil is again used as the initial point with the same target pressure distribution of an RAE 2822 airfoil at $\alpha = 1°$, $M = .75$, and $C_l = .6030$ being specified. It is seen that the final cost function of .005 is lower than that of the final solution for the Hicks-Henne functions. However, not only did this solution take 50 design iterations as opposed to 30 for the first example, but the interim solutions displayed in Figures (2b and 2c) show gross oscillations in the design. The conclusion that may be drawn from this is that the use of B-spline control points localizes the design variables to a point that they act more like using the mesh points themselves as design variables. Thus the high frequency information contained in the control can easily cause oscillations in the design, with the result that there is a greater degree of nonlinearity and a poorer conditioning of the design space as compared to that for the Hicks-Henne functions. A prudent step would be to use a low pass filter when using B-spline control points as design variables to smooth the control in a manner similar to that employed by Jameson [6, 7, 10]. Attempts to increase or decrease the degree and the number of control points for the B-spline curves did not prove to have favorable effects.

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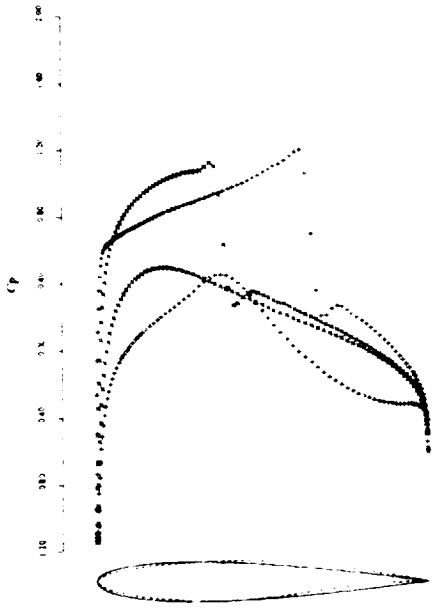
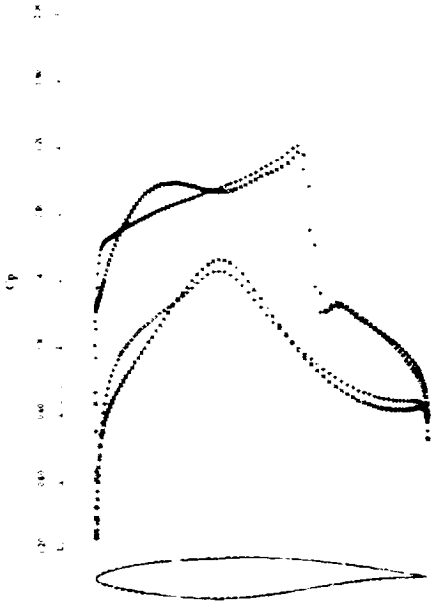
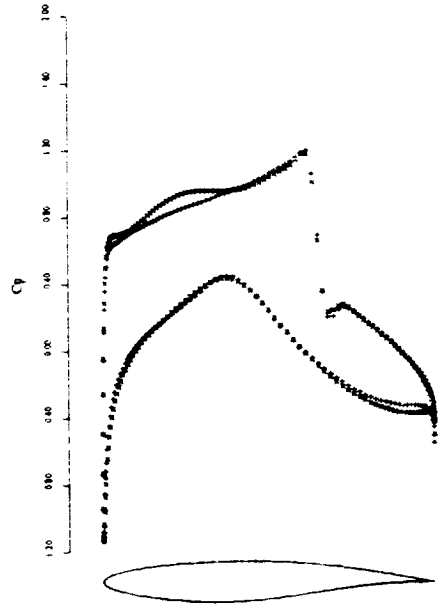


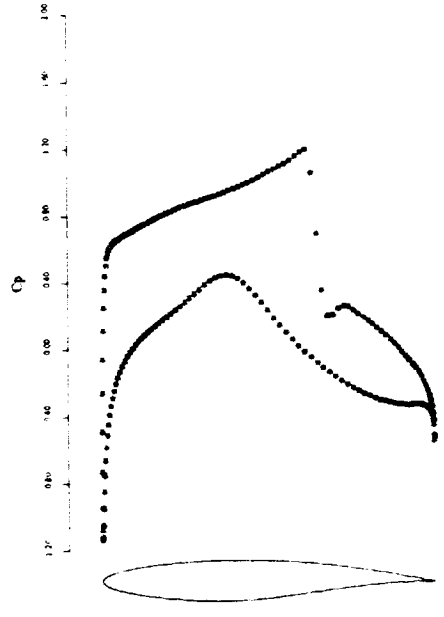
1a: Initial point
Cost function = 68.556



1b Design after 5 iterations
Cost function = 4.247



1c: Design after 10 iterations
Cost function = 0.702

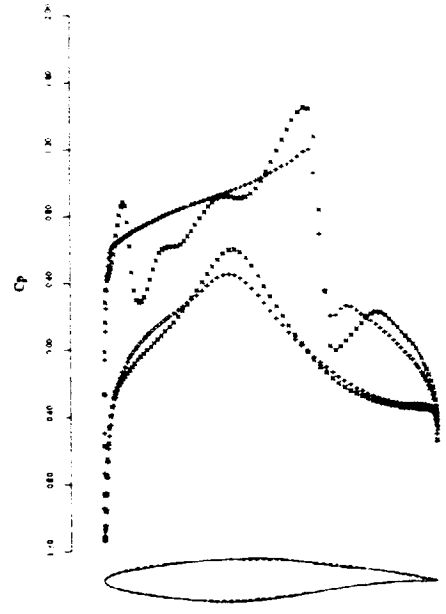


1d: Design after 30 iterations
Cost function = 0.010

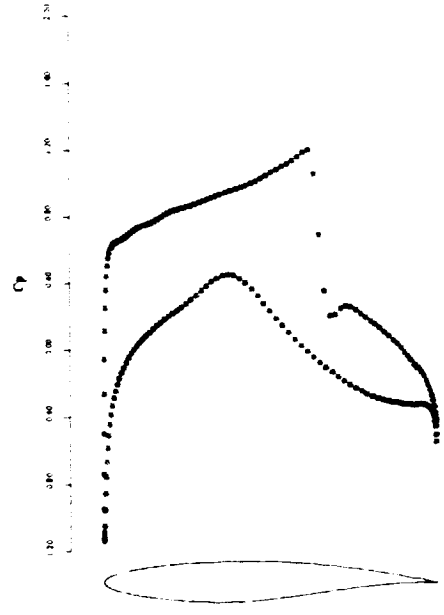
Figure 1: Inverse design from a NACA 0012 to an RAE 2822: **Hicks-Henne design variables**
$M = 0.75$, $C_l = 0.6031$, $C_d = 0.0046$, $\alpha = 1.000$, 50 Design Variables
—, × Initial Airfoil: NACA 0012.
- - -, + Target C_p: RAE, $M = 0.75$.
193x33 Mesh, Euler Solution

2a: Initial point
Cost function = 68.605

2b: Design after 5 iterations
Cost function = 16.500



2c: Design after 10 iterations
Cost function = 10.184



2d: Design after 50 iterations
Cost function = 0.005

Figure 2: Inverse design from a NACA 0012 to an RAE 2822: **B-spline control point design variables**
$M = 0.75$, $C_l = 0.6030$, $C_d = 0.0045$, $\alpha = 1.000$, 50 Design Variables
—, × Initial Airfoil: NACA 0012.
- - -, + Target C_p: RAE, $M = 0.75$.
193x33 Mesh, Euler Solution

END

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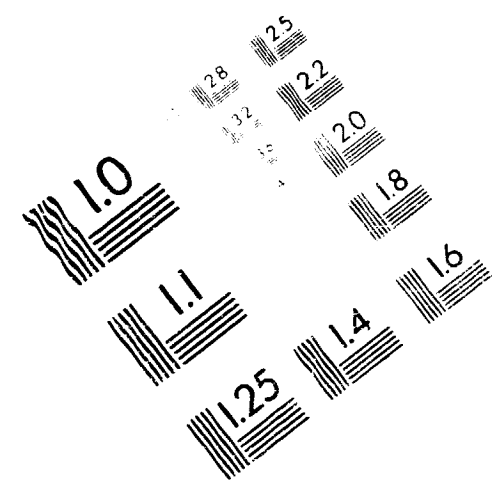

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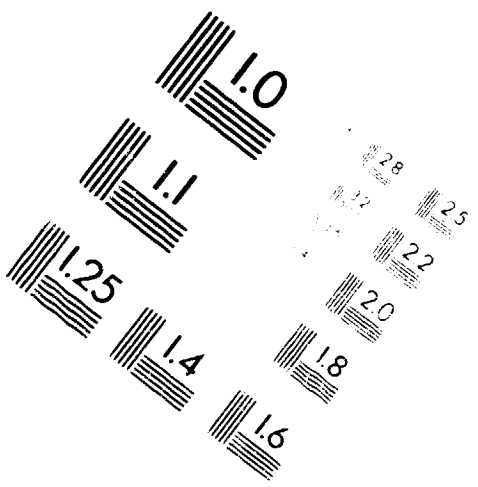

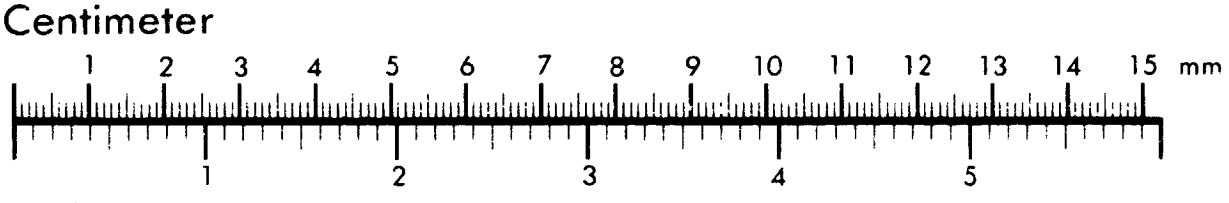

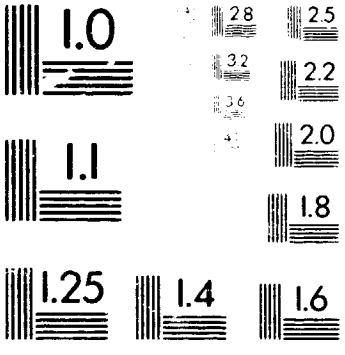

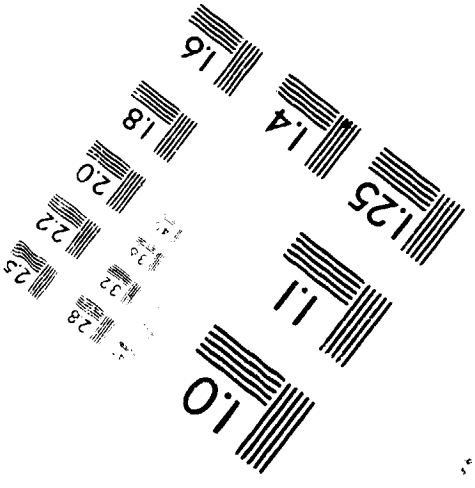

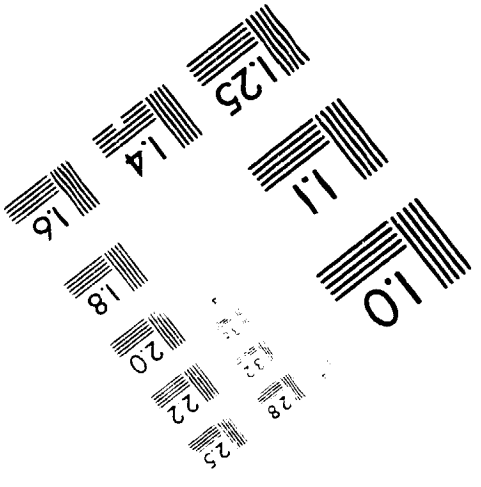

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